

1. A particle P moves in a plane such that its position vector \mathbf{r} metres at time t seconds ($t > 0$) satisfies the differential equation

When $t = 1$, the particle is at the point with position vector $(\mathbf{i} + \mathbf{j})$ m.

(9)



Question 1 continued

Q1

(Total 9 marks)



2. A rocket, with initial mass 1500 kg, including 600 kg of fuel, is launched vertically upwards from rest. The rocket burns fuel at a rate of 15 kg s^{-1} and the burnt fuel is ejected vertically downwards with a speed of 1000 m s^{-1} relative to the rocket. At time t seconds after launch ($t \leq 40$) the rocket has mass m kg and velocity $v \text{ m s}^{-1}$.

(a) Show that

$$\frac{dv}{dt} + \frac{1000}{m} \frac{dm}{dt} = -9.8 \quad (5)$$

(b) Find v at time t , $0 \leq t \leq 40$

(5)



Question 2 continued



Question 2 continued



Question 2 continued

Q2

(Total 10 marks)



P 4 0 1 0 9 A 0 7 2 8

3. A uniform rod PQ , of mass m and length $3a$, is free to rotate about a fixed smooth horizontal axis L , which passes through the end P of the rod and is perpendicular to the rod. The rod hangs at rest in equilibrium with Q vertically below P . One end of a light inextensible string of length $2a$ is attached to the rod at P and the other end is attached to a particle of mass $3m$. The particle is held with the string taut, and horizontal and perpendicular to L , and is then released. After colliding, the particle sticks to the rod forming a body B .

(a) Show that the moment of inertia of B about L is $15ma^2$.

(2)

(b) Show that B first comes to instantaneous rest after it has turned through an angle

$$\arccos\left(\frac{9}{25}\right).$$

(10)



Question 3 continued



Question 3 continued



Question 3 continued

Q3

(Total 12 marks)



4. A body consists of a uniform plane circular disc, of radius r and mass $2m$, with a particle of mass $3m$ attached to the circumference of the disc at the point P . The line PQ is a diameter of the disc. The body is free to rotate in a vertical plane about a fixed smooth horizontal axis, L , which is perpendicular to the plane of the disc and passes through Q . The body is held with QP making an angle β with the downward vertical through Q , where $\sin \beta = 0.25$, and released from rest. Find the magnitude of the component, perpendicular to PQ , of the force acting on the body at Q at the instant when it is released.

[You may assume that the moment of inertia of the body about L is $15mr^2$.]

(6)



Question 4 continued

Q4

(Total 6 marks)



5. The points P and Q have position vectors $4\mathbf{i} - 6\mathbf{j} - 12\mathbf{k}$ and $2\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}$ respectively, relative to a fixed origin O .

Three forces, \mathbf{F}_1 , \mathbf{F}_2 and \mathbf{F}_3 , act along \overrightarrow{OP} , \overrightarrow{QO} and \overrightarrow{QP} respectively, and have magnitudes 7 N, 3 N and $3\sqrt{10}$ N respectively.

- (a) Express \mathbf{F}_1 , \mathbf{F}_2 and \mathbf{F}_3 in vector form. (3)

(b) Show that the resultant of \mathbf{F}_1 , \mathbf{F}_2 and \mathbf{F}_3 is $(2\mathbf{i} - 10\mathbf{j} - 16\mathbf{k})$ N. (2)

(c) Find a vector equation of the line of action of this resultant, giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$, where \mathbf{a} and \mathbf{b} are constant vectors and λ is a parameter. (5)



Question 5 continued



Question 5 continued



Question 5 continued

Q5

(Total 10 marks)



6. A uniform circular pulley, of mass $4m$ and radius r , is free to rotate about a fixed smooth horizontal axis which passes through the centre of the pulley and is perpendicular to the plane of the pulley. A light inextensible string passes over the pulley and has a particle of mass $2m$ attached to one end and a particle of mass $3m$ attached to the other end. The particles hang with the string vertical and taut on each side of the pulley. The rim of the pulley is sufficiently rough to prevent the string slipping. The system is released from rest.

(a) Find the angular acceleration of the pulley.

(8)

When the angular speed of the pulley is Ω , the string breaks and a constant braking couple of magnitude G is applied to the pulley which brings it to rest.

(b) Find an expression for the angle turned through by the pulley from the instant when the string breaks to the instant when the pulley first comes to rest.

(4)



Question 6 continued



Question 6 continued



Question 6 continued

Q6

(Total 12 marks)



7. (a) A uniform lamina of mass m is in the shape of a triangle ABC . The perpendicular distance of C from the line AB is h . Prove, using integration, that the moment of inertia of the lamina about AB is $\frac{1}{6}mh^2$. (7)

(b) Deduce the radius of gyration of a uniform square lamina of side $2a$, about a diagonal. (3)

The points X and Y are the mid-points of the sides RQ and RS respectively of a square $PQRS$ of side $2a$. A uniform lamina of mass M is in the shape of $POQYS$.

- (c) Show that the moment of inertia of this lamina about XY is $\frac{79}{84} Ma^2$. (6)



Question 7 continued



Question 7 continued



Question 7 continued



Question 7 continued



Question 7 continued



Question 7 continued

Q7

(Total 16 marks)

TOTAL FOR PAPER: 75 MARKS

END

